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A STUDY ABOUT SOIL TEMPERATURE VARIATION AT DISTINCT SUB-SURFACES

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ABSTRACT

Heat transfer among sub surfaces of soil has been a diffusion problem of great interest. How heat flow patterns at ground level at deeper surface differ, that is the point of interest and concern to researchers [1]. Heat transfer is closely associated with heat- conductivity, which results from the difference in the temperature of various soil layers. Soil temperature depends upon the process of heat flow between the soil surface and the deeper layer. The flow of heat is directed from warmer layers to cooler layers.Actually soil temperature varies from month to month as a function of incident solar radiation, rainfall, seasonal swings in overlying air temperature, local vegetation cover, type of soil, and depth in the earth, but due to the much higher heat capacity of soil relative to air and the thermal insulation provided by vegetation and surface soil layers, seasonal changes in soil temperature, deep in the ground are much less than and lag significantly behind seasonal changes in overlying air temperature[2]. A few feet below the surface only conductance can take place. So heat transfer by radiation need not be taken into account. As only heat transfer by conductance is to be considered, linearity and time invariance, both the properties will be enjoyed in such situation. It is a matter of experience that on a sunny day, the temperature at the surface of the soil rises rapidly, reaches a maximum in the early afternoon, and falls sharply around sunset. Here in this paper, authors want to elaborate, how the temperature at deeper sub-surfaces will vary and how 24-hour component of the surface temperature is affected at sub-surfaces at specified depths.

KEYWORDS: *Phasor, Propagation Constant, Attenuation Constant, Phase Constant, Attenuation Factor*

INTRODUCTION

To study the soil temperature at a certain depth from the ground and to compare it with surface temperature, consider a vertical column of cross section area A square meters. The thermal resistance, denoted by r,per meter to vertical flow of heat will be,$r = {}^{1}/_{kA}$thohms, where k is thermal conductivity of the soil.As representative value, where k is taken as1.1 $Jm^{-1}s^{-1}K^{-1}$.

(Courtesy of https://www.engineeringtoolbox.com/thermal-conductivity-d_429.html)

A representative value for the specific heat of soil denoted by$s$ is taken as 1000 $JKg^{-1}K^{-1}$.

(Courtesy of https://www.engineeringtoolbox.com/specific-heat-capacity-d_391.html)

The thermal capacitance c per meter will be $c = \rho sA$tharads per meter, where ρdenotes soil density. Here the representative value of ρ is taken as $2000\ kgm^{-3}$.

(Courtesy of https://www.structx.com/Soil_Properties_002.html)

Consider one dimensional diffusion equation$\frac{\partial^2 u}{\partial x^2} = rc\frac{\partial u}{\partial t}$ **(1)**

Where $u(x,t)$ describes the sub-surface temperature (in that vertical column), downward at distance x at time t,taking x as the distance, measured downward from the surface. The gradient of u will be the driving agent causing the changes in u.

Let $u(x,t) = v(x)\theta(t)$ where $v(x)$ and $\theta(t)$ are functions of x and t respectively.

As heat flow will be from warmer surface to cooler surface, here direction of heat flow will be alternating according to whether it is during day or night. The behaviour of temperature and heat flow is quite similar to that of voltage and current in an electrical circuit. So, one can find a very good agreement to utilize concepts of transmission lines in a.c. the current theory of electrical engineering. It would be better to treat here u as temperature phasor with angular frequencyω, that is $u\ (x,t) = Reue^{i\omega t}$. Here Reu represents the real part of phasor u, that is a magnitude of the phasor and$\theta(t) = \ e^{i\omega t}$, where$\omega$ is angular frequency.

Here angular frequency is taken as $\omega = \frac{2\pi}{24}$ rad per hour. Considering day-and-night temperature behavior, one can understand that here time function θwill be sinusoidal.

A phasor is a complex number representing a sinusoidal function whose amplitude A, angular frequency ω, and initial phase are time invariant. It is related to a more general concept called analytic representation which decomposes a sinusoid into the product of a complex constant which encapsulates amplitude and phase dependence is known as a phasor [1],[3].

This means that now the diffusion equation $\frac{\partial^2 u}{\partial x^2} = rc\frac{\partial u}{\partial t}$ converts into an ordinary differential equation,

$$\frac{d^2 v}{dx^2} = rci\omega v \qquad \textbf{(2)}$$

Solving (2), one gets $v(x) = \ c_1 e^{-((i\omega rc)^{1/2})\,x} + c_2 e^{((i\omega rc)^{1/2})\,x}$ **(3)**

Taking$\gamma = (i\omega rc)^{1/2} = \alpha + i\beta$, and solving for α and β,

$$\alpha = \beta = (\frac{\omega rc}{2})^{1/2} \qquad \textbf{(4)}$$

As (3) is a solution of a diffusion equation, diminishing as x increases, $c_2 = 0$.

Substituting $x = 0$, one gets $c_1 = v(0)$.

Thus (3) coverts into

$$v(x) = \ v(0)e^{-\gamma x} = \ v(0)e^{-((i\omega rc)^{1/2})\,x} = v(0)e^{-\alpha x - i\beta x} \Rightarrow v(x) = v(0)\left(e^{-\left(\frac{\omega rc}{2}\right)^{1/2} x}. e^{-i\left(\frac{\omega rc}{2}\right)^{1/2} x}\right) \qquad \textbf{(5)}$$

That is, $v(x) = v(0)\exp(-\alpha x)\exp(-i\beta x)$ **(6)**

Here γ, the propagation constant, is defined as ratio of the complex amplitude of the phasor, at the source of the wave to the complex amplitude at some distance x, such that $\frac{A(0)}{A(x)} = e^{\gamma x}$ [3].

As γ is complex, it can be represented as $\gamma = \alpha + i\beta$, where α, the real part is known as attenuation constant and β, the imaginary part is known as phase constant.

Now temperature function $u(x,t) = v(x)e^{i\omega t} = v(0)\exp(-\alpha x)\exp(i(\omega t - \beta x))$ **(7)**

Thus $\exp(-\alpha x)$, the attenuation factor, will be responsible to decrease amplitude of $u(x,t)$ and the factor $\exp\left(i(\omega t - \beta x)\right)$ implies that the phase of $u(x,t)$ will be retarded by βx radians,

that is, by $\beta x \times \frac{24}{2\pi}$ hours.

Using (6), one can find values of attenuation factor and phase retardation at distinct depths as follows:

As $rc = \frac{\rho s}{k}$, for given values of ρ, s and k, here $rc = \frac{\rho s}{k} = \frac{2000\ kgm^{-3} \times 1000\ Jkg^{-1}\dot{K}^{-1}}{1.1\ Jm^{-1}s^{-1}K^{-1}} = 1.818181818 \times 10^{6} sm^{-2}$

Hence, $\alpha = \beta = \left(\frac{\omega rc}{2}\right)^{1/2} = \left(\frac{2\pi \times 1.81818 \times 10^{6}}{2 \times 24 \times 60 \times 60}\right)^{1/2} = 8.1308644.$

Hence at distinct depths, say at $x = 0.25, 0.50, 0.75, 1.0, 1.25$ meters, the values of attenuation factor and phase retardation will be as follows:

CALCULATIONS

Table 1

Distance(Depth) (in Meters)	Value of αx $(= \beta x) = (8.13086)x$	Attenuation Factor $e^{-\alpha x}$	Phase Retardation $\left(\frac{\beta x \times 24}{2\pi}\right)$ (in hours)
0.25	2.032715	0.130979	7.764399
0.50	4.065430	0.0171556	15.528798
0.75	6.098145	0.0022470	23.293
1.00	8.13086	0.0002943	31.05759
1.25	10.1635805	0.00003855	38.82202

CONCLUSIONS

- Attenuation factor decreases exponentially as depth x increases.
- At ground-surface, that is at $x = 0$, temperature function u itself is not sinusoidal, it possesses harmonics. But as attenuation factor decreases very rapidly, temperature function at sub-surfaces start becoming sinusoidal gradually. At sub-surfaces, downward to surface by more than 0.75 meters, $v(0)$ is multiplied by smaller and smaller factors, which makes u more and more sinusoidal.
- More the depth from the surface, much less, peak to peak variation in temperature at sub-surface compared to that of a 24-hour component of temperature at the surface.

- Phase angle is retarded by a value of βx radians. Here the value of βx is converted in hours.

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